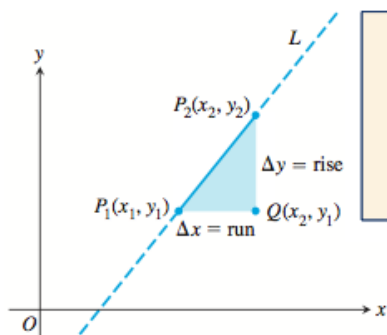


**DEFINITION Increments**

If a particle moves from the point  $(x_1, y_1)$  to the point  $(x_2, y_2)$ , the **increments** in its coordinates are

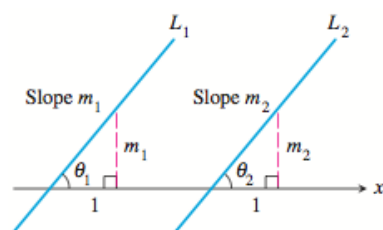
$$\Delta x = x_2 - x_1 \quad \text{and} \quad \Delta y = y_2 - y_1.$$

**DEFINITION Slope**

Let  $P_1(x_1, y_1)$  and  $P_2(x_2, y_2)$  be points on a nonvertical line,  $L$ . The **slope** of  $L$  is

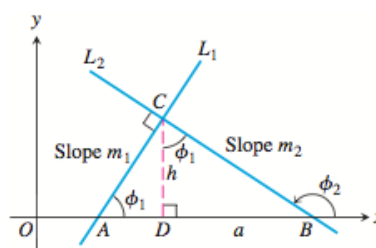
$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

## Parallel Lines



**Figure 1.2** If  $L_1 \parallel L_2$ , then  $\theta_1 = \theta_2$  and  $m_1 = m_2$ . Conversely, if  $m_1 = m_2$ , then  $\theta_1 = \theta_2$  and  $L_1 \parallel L_2$ .

## Perpendicular Lines



**Figure 1.3**  $\triangle ADC$  is similar to  $\triangle CDB$ . Hence  $\phi_1$  is also the upper angle in  $\triangle CDB$ , where  $\tan \phi_1 = a/h$ .

**DEFINITION Point-Slope Equation**

The equation

$$y = m(x - x_1) + y_1$$

is the **point-slope equation** of the line through the point  $(x_1, y_1)$  with slope  $m$ .

**EXAMPLE 3 Using the Point-Slope Equation**

Write the point-slope equation for the line through the point  $(2, 3)$  with slope  $-3/2$ .

**DEFINITION Slope-Intercept Equation**

The equation

$$y = mx + b$$

is the **slope-intercept equation** of the line with slope  $m$  and  $y$ -intercept  $b$ .

**EXAMPLE 4 Writing the Slope-Intercept Equation**

Write the slope-intercept equation for the line through  $(-2, -1)$  and  $(3, 4)$ .

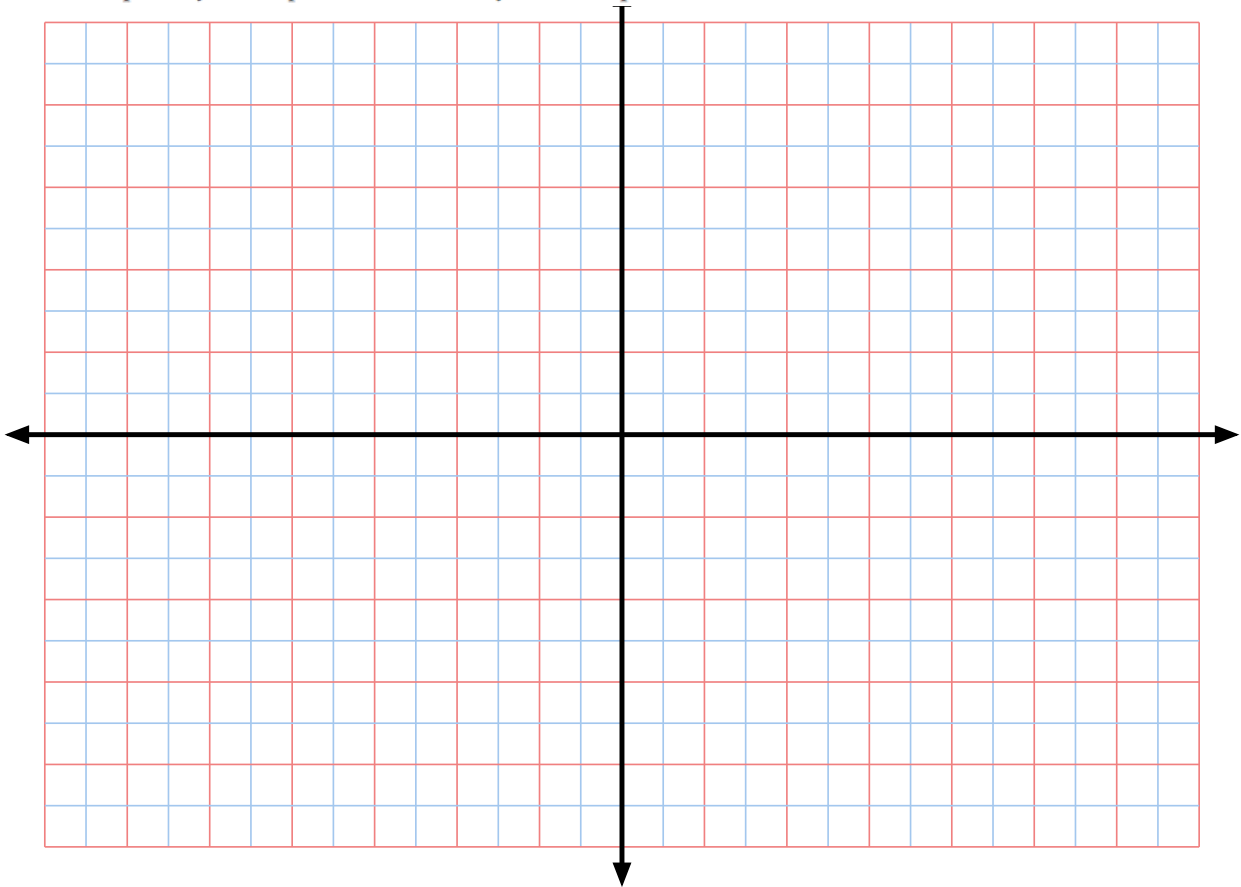
**DEFINITION General Linear Equation**

The equation

$$Ax + By = C \quad (A \text{ and } B \text{ not both } 0)$$

is a **general linear equation** in  $x$  and  $y$ .

Find the slope and y-intercept of the line  $8x + 5y = 20$ . Graph the line.



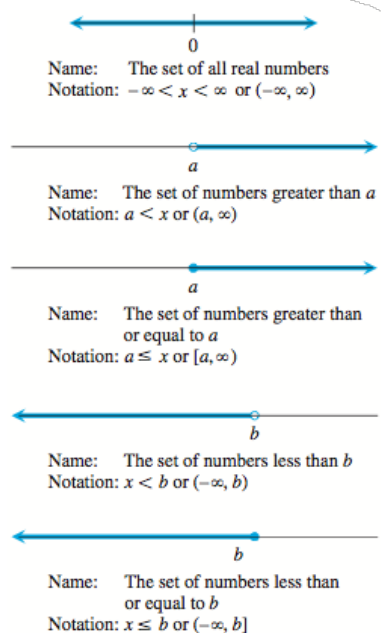
## Functions

Euler invented a symbolic way to say “ $y$  is a function of  $x$ ”:

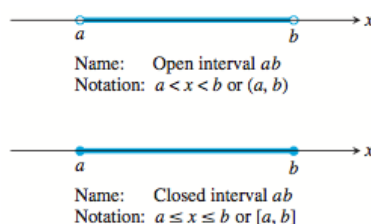
$$y = f(x),$$

### **EXAMPLE 1** The Circle-Area Function

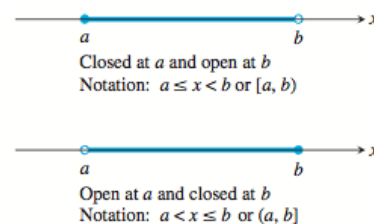
Write a formula that expresses the area of a circle as a function of its radius. Use the formula to find the area of a circle of radius 2 in.



**Figure 1.12** Infinite intervals—rays on the number line and the number line itself. The symbol  $\infty$  (infinity) is used merely for convenience; it does not mean there is a number  $\infty$ .



**Figure 1.10** Open and closed finite intervals.



**Figure 1.11** Half-open finite intervals.

The endpoints of an interval make up the interval's **boundary** and are called **boundary points**. The remaining points make up the interval's **interior** and are called **interior points**. **Closed intervals** contain their boundary points. **Open intervals** contain no boundary points. Every point of an open interval is an interior point of the interval.



**EXAMPLE 2** Identifying Domain and Range of a Function

Identify the domain and range, and then sketch a graph of the function.

(a)  $y = \frac{1}{x}$       (b)  $y = \sqrt{x}$

**Graph Viewing Skills**

1. Recognize that the graph is reasonable.
2. See all the important characteristics of the graph.
3. Interpret those characteristics.
4. Recognize grapher failure.

**EXAMPLE 3 Identifying Domain and Range of a Function**

Use a grapher to identify the domain and range, and then draw a graph of the function.

(a)  $y = \sqrt{4 - x^2}$

(b)  $y = x^{2/3}$

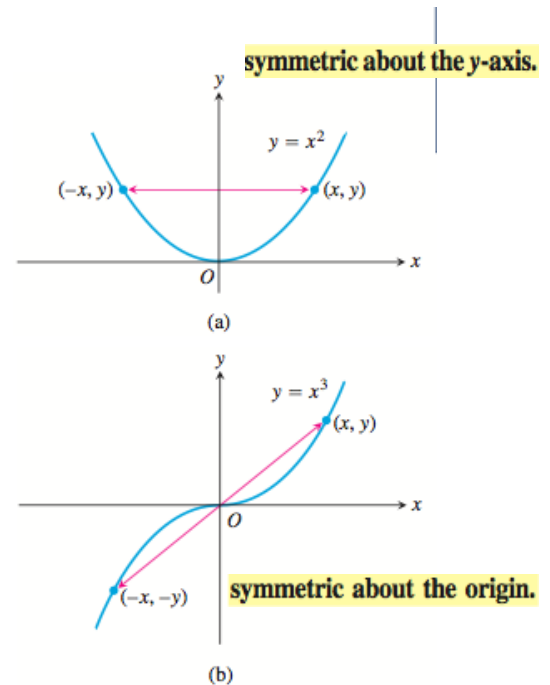
**DEFINITIONS Even Function, Odd Function**

A function  $y = f(x)$  is an

**even function of  $x$**  if  $f(-x) = f(x)$ ,

**odd function of  $x$**  if  $f(-x) = -f(x)$ ,

for every  $x$  in the function's domain.



**EXAMPLE 4**

$$f(x) = x^2$$

$$f(x) = x^2 + 1$$

$$f(x) = x$$

$$f(x) = x + 1$$

**EXAMPLE 5 Graphing Piecewise-Defined Functions**

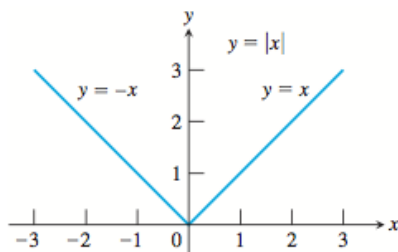
Graph  $y = f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ 1, & x > 1. \end{cases}$

## Absolute Value Function

The **absolute value function**  $y = |x|$  is defined piecewise by the formula

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0. \end{cases}$$

The function is even, and its graph (Figure 1.19) is symmetric about the y-axis.



**Figure 1.19** The absolute value function has domain  $(-\infty, \infty)$  and range  $[0, \infty)$ .

### EXAMPLE 7 Using Transformations

Draw the graph of  $f(x) = |x - 2| - 1$ . Then find the domain and range.

## Composite Functions

### EXAMPLE 8 Composing Functions

Find a formula for  $f(g(x))$  if  $g(x) = x^2$  and  $f(x) = x - 7$ . Then find  $f(g(2))$ .

### EXPLORATION 1 Composing Functions

Some graphers allow a function such as  $y_1$  to be used as the independent variable of another function. With such a grapher, we can compose functions.

1. Enter the functions  $y_1 = f(x) = 4 - x^2$ ,  $y_2 = g(x) = \sqrt{x}$ ,  $y_3 = y_2(y_1(x))$ , and  $y_4 = y_1(y_2(x))$ . Which of  $y_3$  and  $y_4$  corresponds to  $f \circ g$ ? to  $g \circ f$ ?
2. Graph  $y_1$ ,  $y_2$ , and  $y_3$  and make conjectures about the domain and range of  $y_3$ .
3. Graph  $y_1$ ,  $y_2$ , and  $y_4$  and make conjectures about the domain and range of  $y_4$ .
4. Confirm your conjectures algebraically by finding formulas for  $y_3$  and  $y_4$ .

# Section 1.3

Grade: «grade»  
Subject: «subject»  
Date: «date»



**DEFINITION Exponential Function**

Let  $a$  be a positive real number other than 1. The function

$$f(x) = a^x$$

is the **exponential function with base  $a$** .

**EXAMPLE 1 Graphing an Exponential Function**

Graph the function  $y = 2(3^x) - 4$ . State its domain and range.

### **EXAMPLE 2** Finding Zeros

Find the zeros of  $f(x) = 5 - 2.5^x$  graphically.

### Rules for Exponents

If  $a > 0$  and  $b > 0$ , the following hold for all real numbers  $x$  and  $y$ .

1.  $a^x \cdot a^y = a^{x+y}$
2.  $\frac{a^x}{a^y} = a^{x-y}$
3.  $(a^x)^y = (a^y)^x = a^{xy}$
4.  $a^x \cdot b^x = (ab)^x$
5.  $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

**Table 1.7** United States Population

Year	Population (millions)	Ratio
1998	276.1	
		$279.3/276.1 \approx 1.0116$
1999	279.3	
		$282.4/279.3 \approx 1.0111$
2000	282.4	
		$285.3/282.4 \approx 1.0102$
2001	285.3	
		$288.2/285.3 \approx 1.0102$
2002	288.2	
		$291.0/288.2 \approx 1.0097$
2003	291.0	

Source: Statistical Abstract of the United States, 2004–2005.

### EXAMPLE 3 Predicting United States Population

Use the data in Table 1.7 and an exponential model to predict the population of the United States in the year 2010.

*continues*

**DEFINITIONS Exponential Growth, Exponential Decay**

The function  $y = k \cdot a^x$ ,  $k > 0$  is a model for **exponential growth** if  $a > 1$ , and a model for **exponential decay** if  $0 < a < 1$ .

Year	Population (millions)
1880	50.2
1890	63.0
1900	76.2
1910	92.2
1920	106.0
1930	123.2
1940	132.1
1950	151.3
1960	179.3
1970	203.3
1980	226.5
1990	248.7

Source: *The Statistical Abstract of the United States, 2004–2005.*

Use exponential regression

**EXAMPLE 5 Predicting the U.S. Population**

Use the population data in Table 1.8 to estimate the population for the year 2000. Compare the result with the actual 2000 population of approximately 281.4 million.

**EXAMPLE 6** Interpreting Exponential Regression

What *annual* rate of growth can we infer from the exponential regression equation in Example 5?